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Final Exam, MTH 213, Spring 2019

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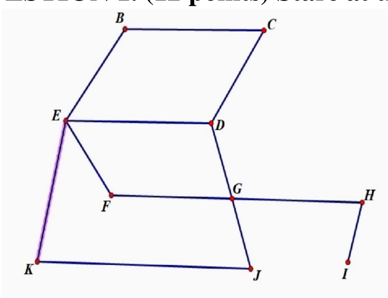
SCORE = $\frac{}{78}$ ($\square$ (MW) $\square$ (UTR)

QUESTION 1. (12 points) Stare at the following graph, say G.



(i) Is G a bipartite graph? If yes, then redraw G as a bipartite graph. If no, then explain

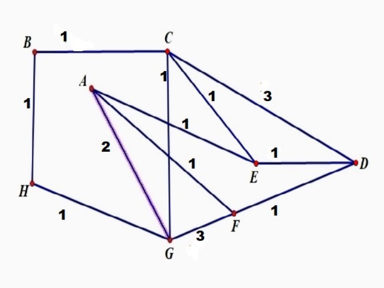
(ii) Is G an Euler trail (path) that is not an Euler Circuit? If yes write down such trail. If no, then explain.

(iii) Let F be a spanning subgraph of G such that F is a tree. Draw F.

(iv) Let F be an induced subgraph of G. Suppose that F is a tree and F has more than 6 vertices. Draw F

QUESTION 2. **(4 points)** (SHOW STEPS) Let $d = gcd(117, 96)$. Find d, then find a, b such that $d = 117a + 96b$.

QUESTION 3. **(5 points)** Stare at the following graph



A Post office is located at vertex A. The MAIL man needs to visit each block (vertex) exactly once and then come back to vertex A.

a) Write down all possible Hamiltonian-Cycles starting from A and calculate the weight of each cycle.

b) Find the minimum weight-cycle that he should use.

QUESTION 4. (6 points) (SHOW STEPS) Let X be the number of laptops in a particular storage. Given $216 < X < 288$, $X \equiv 6 \ (mod\, 9)$ and $X \equiv 5 \ (mod\, 8)$. Find the value of X.

QUESTION 5. (3 points) There are 823 balls. Each ball is either RED or BLUE OR GREEN OR BLACK OR YELLOW. An even number, say n, of balls are selected randomly. What is the minimum value of n that will secure at least 83 balls are of the same color. SHOW the work

QUESTION 6. (Show steps)

(i) **(2 points)** Let n be an even number. Use direct proof and convince me that $5n^2 + 11$ is an odd integer

(ii) **(3 points)** Given G is a complete bipartite graph with m vertices. The degrees of the vertices in DESCENDING order are the following integers: $a_1 = 5, a_2, a_3 = 5, a_4, a_5, ..., a_m$ such that $a_4 \neq 5$.

a. Find the value of m. Find the values of $a_2, a_4, a_5, ..., a_m$ (i.e., find the degrees of all vertices).

b. DRAW such graph

(iii) **(3 points)** Find all possible values of X over Planet Z where $6X \equiv 9 \ (mod\ 15)$.

QUESTION 7. (**6 points**)) Consider the following code

For $k = 3$ to $(n+2)$
$S = k^3 + 3 * k^2 + K - 2$
For $i = 1 \ to \ k$
$L = k^2 + 7 * k - 3$
Next i
For $m = 4$ to $(5k+3)$
$H = 9 * m^2 + 6 * m - 3$
next m
next k

(i) Find the exact number of addition, subtraction, multiplication that the code executed.

(ii) Find the complexity of the code.

QUESTION 8. (**4 points**) There are 929 available odd integers. Then there are at least n odd integers, say $a_1, a_2, ..., a_n$ such that $a_1 \ (mod\ 8) = a_2 \ (mod\ 8) = ... = a_n \ (mod\ 8)$. Find the maximum value of n. SHOW THE WORK.

QUESTION 9. (**3 points**) $f(x) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 & 10 & 9 \end{pmatrix}$. Find the minimum number n such that $f^n = f \ o \ f \cdots \ o \ f$ (n-times) $= I(x)$, where $I(x)$ is the identity map, i.e., $I(x) = x$ for every $x \in \{1, 2, 3, ..., 10\}$.

QUESTION 10. (4 points) Let $f : (-\infty, a] \to [b, \infty)$ such that $a \neq 0$ and $f(x) = x^2 - 9$ is a bijective function. Find a value of a and a value of b. Then find $f^{-1}(x)$ and find its domain and codomain.

QUESTION 11. (6 points) Let $A = \{10, 11, 12, 13, 14, 16\}$ Define "$\leq$" on A such that $a" \leq "b$ if and only if $(a - b) \in \{0, 2, 3, 4, 5, 6\}$. Then "$\leq$" is a partial order relation on A. Then

(i) Draw the Hasse diagram of such relation.

(ii) Find $13 \vee 14$

(iii) Find $10 \wedge 11$

(iv) Find $12 \wedge 11$

QUESTION 12. (6 points) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 11, 19, 21, 27, 29, 30, 31, 33, 37\}$. Define "=" on A such that $a" = "b$ if and only if $(2a + 4b)\ (mod\ 6) = 0$. Then "=" is an equivalence relation. (hint: note that $(2a + 4b)\ (mod\ 6) = (2a - 2b)\ (mod\ 6)$ since $-2 (mod\ 6) = 4$)

1) Find all equivalence classes of A.

2) If we view "=" as a subset of $A \times A$, how many elements does "=" have?

QUESTION 13. (**3 points**) Use the four-method and convince me that $\sqrt{34}$ is irrational number.

QUESTION 14. (1)(**4 points**) Use math-induction and convince me that $3 \mid (n^3 + 5n)$ for every integer $n \geq 1$. (note that $(n+1)^3 = n^3 + 3n^2 + 3n + 1$)

(2) (**4 points**) Use math-induction and convince me that $n! > 4^{(n-2)}$ for every integer $n \geq 3$ (note that $n!$ is the normal n factorial, i.e., $n! = n(n-1)(n-2)\cdots 2 \cdot 1$)

Faculty information

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